# INTERNATIONAL STANDARD

ISO 11146-1

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Lasers and laser-related equipment — Test methods for laser beam widths, divergence angles and beam propagation ratios —

Part 1: Stigmatic and simple astigmatic beams

Lasers et équipements associés aux lasers — Méthodes d'essai des largeurs du faisceau, angles de divergence et facteurs de limite de diffraction —

Partie 1: Faisceaux stigmatiques et astigmatiques simples



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### Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

ISO 11146-1 was prepared by Technical Committee ISO/TC 172, *Optics and photonics*, Subcommittee SC 9, *Electro-optical systems*.

This first edition of ISO 11146-1, together with ISO/TR 11146-3 cancels and replaces ISO 11146:1999, the contents of which have been technically revised and augmented.

ISO 11146 consists of the following parts, under the general title *Lasers and laser-related equipment* — *Test methods for laser beam widths, divergence angles and beam propagation ratios*:

- Part 1: Stigmatic and simple astigmatic beams
- Part 2: General astigmatic beams
- Part 3: Intrinsic and geometrical laser beam classification, propagation and details of test methods (Technical Report)

### Introduction

The propagation properties of every laser beam can be characterized within the method of second order moments by ten independent parameters (see ISO/TR 11146-3). However, due to their higher symmetry most laser beams of practical interest need fewer parameters for a complete description. Most lasers of practical use emit beams which are stigmatic or simple astigmatic because of their resonator design.

This part of ISO 11146 describes the measurement methods for stigmatic and simple astigmatic beams while Part 2 deals with the measurement procedures for general astigmatic beams. For beams of unknown type the methods of Part 2 shall be applied. Beam characterization based on the method of second order moments as described in both parts is only valid within the paraxial approximation.

The theoretical description of beam characterization and propagation as well as the classification of laser beams is given in ISO/TR 11146-3, which is an informative Technical Report and describes the procedures for background subtraction and offset correction.

In this part of ISO 11146, the second order moments of the power (energy) density distribution are used for the determination of beam widths. However, there may be problems experienced in the direct measurement of these quantities in the beams from some laser sources. In this case, other indirect methods of the measurement of the second order moments may be used as long as comparable results are achievable.

In ISO/TR 11146-3, three alternative methods for beam width measurement and their correlation with the method used in this part of ISO 11146 are described. These methods are:

- variable aperture method;
- moving knife-edge method;
- moving slit method.

The problem of the dependence of the measuring result on the truncation limits of the integration area has been investigated and evaluated by an international round robin experiment carried out in 1997. The results of this round robin testing were taken into consideration during the preparation of this document.

The International Organization for Standardization (ISO) draws attention to the fact that it is claimed that compliance with this document may involve the use of a patent concerning the determination of beam characteristics by measuring along the beam caustic of the transformed beam produced by a lens as described in 5.5.

ISO takes no position concerning the evidence, validity and scope of this patent right.

The holder of this patent right (U.S. No. 5,267,012) has assured ISO that he is willing to negotiate licences under reasonable and non-discriminatory terms and conditions with applicants throughout the world. In this respect, the statement of the holder of this patent right is registered with the ISO. Information may be obtained from:

Coherent Inc. 5100 Patrick Henry Drive Santa Clara, CA 95056-0980 USA

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights other than those identified above. ISO shall not be held responsible for identifying any or all such patent rights.

# Lasers and laser-related equipment — Test methods for laser beam widths, divergence angles and beam propagation ratios —

## Part 1: Stigmatic and simple astigmatic beams

### 1 Scope

This part of ISO 11146 specifies methods for measuring beam widths (diameter), divergence angles and beam propagation ratios of laser beams. This part of ISO 11146 is only applicable for stigmatic and simple astigmatic beams. If the type of the beam is unknown, and for general astigmatic beams, ISO 11146-2 should be applied.

### 2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 11145, Optics and optical instruments — Lasers and laser-related equipment — Vocabulary and symbols

ISO 11146-2, Lasers and laser-related equipment — Test methods for laser beam widths, divergence angles and beam propagation ratios — Part 2: General astigmatic beams

ISO 13694, Optics and optical instruments — Lasers and laser-related equipment — Test methods for laser beam power (energy) density distribution

IEC 61040:1990, Power and energy measuring detectors, instruments and equipment for laser radiation

### 3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 11145, ISO 13694, IEC 61040 and the following apply.

NOTE The *x*-, *y*- and *z*-axes in these definitions refer to the laboratory system as described in Clause 4. Here and throughout this document the term "power density distribution E(x,y,z)" refers to continuous wave sources. It might be replaced by "energy density distribution H(x,y,z)" in case of pulsed sources.

### 3.1

### first order moments of a power density distribution

 $\overline{x}, \overline{y}$ 

centroid coordinates of the power density distribution of a cross section of a beam given as

$$\overline{x}(z) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) x \, dx \, dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) dx \, dy}$$
(1)

1

and

$$\overline{y}(z) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) y \, dx \, dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) dx \, dy}$$
(2)

NOTE For practical application, the infinite integration limits are reduced in a specific manner as given in Clause 7.

### 3.2

### second order moments of a power density distribution $\sigma_{x}^{2},\sigma_{y}^{2},\sigma_{xy}^{2}$

normalized weighted integrals over the power density distribution, given as

$$\sigma_{\mathbf{x}}^{2}(z) = \left\langle x^{2} \right\rangle = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z)(x - \overline{x})^{2} dx dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) dx dy}$$
(3)

and

$$\sigma_{\mathbf{y}}^{2}(z) = \left\langle y^{2} \right\rangle = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z)(y - \overline{y})^{2} dx dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) dx dy}$$
(4)

and

$$\sigma_{xy}^{2}(z) = \langle xy \rangle = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z)(x - \overline{x})(y - \overline{y}) dx dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) dx dy}$$
(5)

NOTE 1 For practical application, the infinite integration limits are reduced in a specific manner as given in Clause 7.

NOTE 2  $\sigma_{xy}^2(z)$  is a symbolic notation, and not a true square. This quantity can take positive, negative or zero value.

NOTE 3 The angular brackets are the operator notations as used in ISO 11146-2 and ISO/TR 11146-3.

### 3.3

### principal axes of a power density distribution

axes of the maximum and minimum beam extent based on the centered second order moments of the power density distribution in a cross section of the beam

NOTE The axes of maximum and minimum extent are always perpendicular to each other.

### 3.4

### orientation of a power density distribution

φ

angle between the *x*-axis of the laboratory system and that of the principal axis of the power density distribution which is closer to the *x*-axis

NOTE From this definition it follows that  $-\pi/4 < \varphi < \pi/4$  for  $|\varphi| \neq \pi/4$ ; if  $\varphi = \pm \pi/4$ ,  $\varphi$  is defined as the angle between the *x*-axis and the major principal axis (axis of maximum extent) of the power density distribution.

### 3.5

### beam widths

 $d_{\sigma x}, d_{\sigma v}$ 

extent of a power density distribution in a cross section of the beam at an axial location z along that principal axis which is closer to the x- or y-axis of the laboratory coordinate system, respectively, based on the centered second order moments of the power density distribution

NOTE 1 If the principal axes make the angle  $\pi/4$  with the *x*- and *y*-axes of the laboratory coordinate system, then  $d_{\alpha}$  is by convention the larger beam width.

NOTE 2 This definition differs from that given in ISO 11145:2001, subclause 3.5.2, where the beam widths are defined only in the laboratory system, whereas for the purposes of this part of ISO 11146 the beam widths are defined in the principal axes system.

### 3.6

### ellipticity of a power density distribution

Е

ratio between the minimum and maximum beam widths

### 3.7

### circular power density distribution

power density distribution having an ellipticity greater than 0,87

### 3.8

### beam diameter

 $d_{\sigma}$ 

extent of a circular power density distribution, based on the second order moments

### 3.9

### stigmatism

property of a beam having circular power density distributions in any plane under free propagation and showing power density distributions after propagation through a cylindrical lens all having the same or azimuthal orientation as that lens

### 3.10

### simple astigmatism

property of a non-stigmatic beam whose azimuth angle shows a constant orientation under free propagation, and which retains its original orientation after passing through a cylindrical optical element whose cylindrical axis is parallel to one of the principal axes of the beam

NOTE The principal axes of a power density distribution corresponding to a beam with simple astigmatism are called the principal axes of that beam.

### 3.11

### general astigmatism

property of a beam which is neither stigmatic nor simple astigmatic

NOTE This part of ISO 11146 deals only with stigmatic and simple astigmatic beams. Refer to ISO 11146-2 for general astigmatic beams.

### 3.12

### beam waist locations

 $z_{0x}, z_{0y}, z_0$ 

positions from z = 0 reference plane where the beam widths or the beam diameter reach their minimum values along the axis of propagation

See Figure 1.

NOTE 1 In the case of general astigmatic beams, which are outside the scope of this part of the standard, this definition does not apply.

NOTE 2 For simple astigmatic beams the waist locations  $z_{0x}$  and  $z_{0y}$  corresponding to the principal axes, may or may not coincide.

### 3.13

### beam waist widths

 $d_{\sigma x0}$ ,  $d_{\sigma y0}$  beam widths at the beam waist locations of a simple astigmatic beam

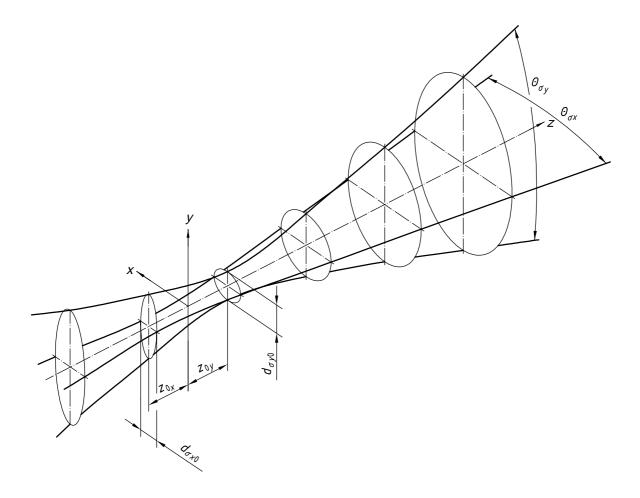
 $d_{\sigma x0}$  is the beam width  $d_{\sigma x}$  at location  $z_{0x}$ ,  $d_{\sigma y0}$  is the beam width  $d_{\sigma y}$  at location  $z_{0y}$ . NOTE

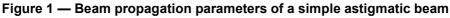
### 3.14

### beam waist diameter

 $d_{\sigma 0}$ 

beam diameter at the beam waist location of a stigmatic beam





### 3.15 beam divergence angles

 $\Theta_{\sigma x}, \ \Theta_{\sigma y}, \ \Theta_{\sigma}$ 

measure for the increase of the beam widths or beam diameter with increasing distance from the beam waist locations, given by

$$\Theta_{\sigma x} = \lim_{(z-z_{0x})\to\infty} \frac{d_{\sigma x}(z)}{z-z_{0x}}$$
(6)

and

$$\Theta_{\sigma y} = \lim_{(z-z_{0y})\to\infty} \frac{d_{\sigma y}(z)}{z-z_{0y}}$$
(7)

for simple astigmatic beams and

$$\Theta_{\sigma} = \lim_{(z-z_0) \to \infty} \frac{d_{\sigma}(z)}{z-z_0}$$
(8)

for stigmatic beams

NOTE 1 The beam divergence is expressed as a full angle.

NOTE 2 This definition differs from that given in ISO 11145:2001, subclause 3.18.2, where the beam divergence angles are defined only in the laboratory system, whereas for the purposes of this part of ISO 11146 the beam divergence angles are defined in the principal axes system.

#### 3.16 beam propagation ratios

NOTE 1 The term "beam propagation ratio" replaces "times-diffraction-limit factor" which was used in ISO 11146:1999.

NOTE 2 Beam propagation ratios, as defined in 3.16.1 and 3.16.2, are propagation invariants for stigmatic and simple astigmatic beams, only as long as the optics involved do not change the stigmatic or the simple astigmatic character of the beam.

### 3.16.1 beam propagation ratios

 $M_{x}^{2}$  and  $M_{y}^{2}$ 

 $\langle simple astigmatic beams \rangle$  ratios of the beam parameter product along the principal axes of the beam of interest to the beam parameter product of a diffraction-limited, perfect Gaussian beam of the same wavelength  $\lambda$ 

$$M_{x}^{2} = \frac{\pi}{\lambda} \quad \frac{d_{\sigma x 0} \Theta_{\sigma x}}{4}$$

$$M_{y}^{2} = \frac{\pi}{\lambda} \quad \frac{d_{\sigma y 0} \Theta_{\sigma y}}{4}$$
(9)
(10)

### 3.16.2 beam propagation ratio

 $M^2$ 

 $\langle stigmatic beams \rangle$  ratio of the beam parameter product of the beam of interest to the beam parameter product of a diffraction-limited, perfect Gaussian beam (TEM<sub>00</sub>) of the same wavelength  $\lambda$ 

$$M^{2} = \frac{\pi}{\lambda} \quad \frac{d_{\sigma 0}\Theta_{\sigma}}{4} \tag{11}$$

### 4 Coordinate systems

The *x*-, *y*- and *z*-axes define the orthogonal space directions in the laboratory axes system and shall be specified by the user. The *z*-axis shall coincide approximately with the direction of the beam. The *x*- and *y*-axes are transverse axes, usually horizontal and vertical, respectively. The origin of the *z*-axis is in a reference *x*-*y* plane defined by the manufacturer, e.g. the front of the laser enclosure.

### 5 Test principles

### 5.1 Applicability

The following test principles are only valid for stigmatic and simple astigmatic beams. For general astigmatic beams ISO 11146-2 shall be applied.

### 5.2 Beam widths and beam diameter

For the determination of beam widths or diameter at location z, the power density distribution of the laser beam shall be measured in the x-y plane at this location z. Suitable background correction shall be applied to the measured data if necessary (refer to ISO/TR 11146-3). From the measured power density distribution the first order and centered second order moments are calculated. From the centered second order moments the beam widths,  $d_{\sigma x}(z)$ ,  $d_{\sigma y}(z)$ , the ellipticity,  $\varepsilon$ , and, if appropriate, the beam diameter,  $d_{\sigma}(z)$ , are to be determined.

### 5.3 Beam divergence angles

The determination of the divergence angles follows from measurements of the beam widths or the beam diameter in the focal plane of a focusing element.

First, the laser beam shall be transformed by an aberration-free focusing element. For a simple astigmatic beam, the beam widths  $d_{\sigma xf}$  and  $d_{\sigma yf}$  are measured one focal length, f, away from the rear principal plane of the focusing element. The corresponding divergence angles  $\Theta_{\sigma x}$  and  $\Theta_{\sigma y}$  are determined using the relationships

$$\Theta_{\sigma x} = \frac{d_{\sigma x f}}{f}$$
(12)

and

$$\Theta_{\sigma y} = \frac{d_{\sigma yf}}{f}$$
(13)

For stigmatic beams, the beam diameter  $d_{\it of}$  is measured and the divergence angle  $\varTheta_{\sigma}$  is determined by using

$$\Theta_{\sigma} = \frac{d_{\sigma f}}{f} \tag{14}$$

### 5.4 Beam propagation ratios

For the determination of the beam propagation ratios  $M_x^2$ ,  $M_y^2$  or  $M^2$ , it is necessary to determine the beam waist widths  $d_{\sigma x0}$ ,  $d_{\sigma y0}$  or the waist diameter  $d_{\sigma 0}$  and the related beam divergence angles  $\Theta_{\sigma x}$ ,  $\Theta_{\sigma y}$  or  $\Theta_{\sigma}$ .

### 5.5 Combined measurement of beam waist locations, beam widths, beam divergence angles and beam propagation ratios

The beam widths data along the propagation axis shall be fitted to a hyperbola as discussed in Clause 9. The beam waist locations, beam waist widths, beam divergence angles and beam propagation ratios are derived from the fit parameters.

### 6 Measurement arrangement and test equipment

### 6.1 General

The test is based on the measurement of the cross-sectional power density distribution at a number of axial locations along the beam propagation axis.

### 6.2 Preparation

The optical axis of the measuring system should be coaxial with the laser beam to be measured. Suitable optical alignment devices are available for this purpose (e.g. aligning lasers or steering mirrors).

The aperture of the optical system should accommodate the entire cross-section of the laser beam. Losses by clipping shall be smaller than 1 % of the total beam power or energy. In order to test this, apertures of different widths can be introduced into the beam path in front of each optical component. The aperture which reduces the output signal by 5 % should have a diameter less than 0,8 times the aperture of the optical component.

The attenuators or beam-forming optics should be mounted such that the optical axis runs through the geometrical centres. Care shall be taken to avoid systematic errors. Reflections, interference effects, external ambient light, thermal radiation or air draughts are all potential sources of increased uncertainty.

### 6.3 Control of environment

Suitable measures such as mechanical and acoustical isolation of the test set-up, shielding from extraneous radiation, temperature stabilization of the laboratory, choice of low-noise amplifiers shall be taken to ensure that the contribution to the total probable uncertainty of the parameters to be measured is low.

Care should be taken to ensure that the atmospheric environment in high-power laser beam paths does not contain gases or vapours that can absorb the laser radiation and cause thermal distortion in the beam to be measured.

### 6.4 Detector system

Measurement of the cross-sectional power density distribution requires the use of a detection system with high spatial resolution and high signal-to-noise ratio.

The accuracy of the measurement is directly related to the spatial resolution of the detector system and its signal-to-noise ratio. The latter is important for laser beams with low power densities at larger diameters (e.g. for diffracted parts of the laser beams).

NOTE For pixel based detector systems, the spatial resolution should be at least 1/20 of the beam width or beam diameter.

In practice, noise in the wings of the power density distribution E(x,y,z) may readily dominate the second order moment integral. Thus, it is usually necessary to apply background correction procedures. Refer to ISO/TR 11146-3 for further details.

The radiation detector system shall be in accordance with IEC 61040:1990, in particular with Clauses 3 and 4. Furthermore, the following points shall be noted.

- Care shall be taken to ascertain the damage thresholds of the detector surface so that they are not
  exceeded by the laser beam.
- It shall be confirmed, from manufacturers' data or by measurement, that the output quantity of the detector system (e.g. the voltage) is linearly dependent on the input quantity (laser power). Any wavelength dependency, non-linearity or non-uniformity of the detector or the electronic device shall be minimized or corrected by use of a calibration procedure.
- When using a scanning device for determining the power density distribution, care shall be taken to ensure that the laser output is spatially and temporally stable during the whole scanning period.
- When measuring pulsed laser beams, the trigger time delay of sampling as well as the measuring time interval play an important role because the beam parameters may change during the pulse. Therefore it is necessary to specify these parameters in the test report.

### 6.5 Beam-forming optics and optical attenuators

If the beam cross-sectional area is greater than the detector area, a suitable optical system shall be used to reduce the beam cross-sectional area on the detector surface. The change in magnification shall be taken into account during the evaluation procedure.

Optics shall be selected appropriate to wavelength.

An attenuator may be required to reduce the laser power density on the surface of the detector.

Optical attenuators shall be used when the laser output-power or power density exceeds the detector's working (linear) range or the damage threshold. Any wavelength, polarization and angular dependency, non-linearity or non-uniformity, including thermal effects of the optical attenuator, shall be minimized or corrected by use of a calibration procedure.

None of the optical elements used shall significantly influence the relative power density distribution.

### 6.6 Focusing system

The focusing system shall conform to the requirements relating to the beam-forming optics given in 6.5. The total uncertainty contributed by the focusing system shall be less than 1 % of the beam width.

### 7 Beam widths and beam diameter measurement

#### 7.1 Test procedure

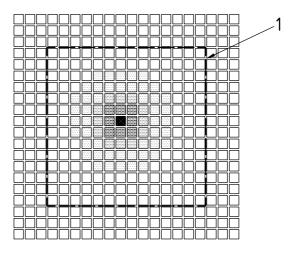
Before the measurements are started, the laser shall be warmed up for at least 1 h (unless otherwise stated by the manufacturer) in order to achieve thermal equilibrium. The measurements shall be carried out at the operating conditions specified by the laser manufacturer for the type of laser being evaluated.

Repeat at least five times the measurement of the cross-sectional power density distribution at each location z at which the beam widths are determined.

### 7.2 Evaluation

Before evaluating beam widths or beam diameters, background correction procedures shall be applied to the measured distribution (refer to ISO/TR 11146-3).

The first-order moments and second-order moments are calculated from the measured and corrected distributions. The corresponding integrations are carried out on a subset of the measured data, called the integration area (see Figure 2), because otherwise noise in the data may dominate the integrals. In many cases a proper choice of the integration area is crucial for reliable results. The following procedure relates the size and position of the integration area to the size and position of the measured power density distribution, which are initially unknown. Hence, an iterative procedure follows.



### Key

1 outline of integration area

#### Figure 2 — Detection array with schematic representation of the laser beam and the integration area

All integrations in equations (1) to (5) are performed on an integration area which is centered to the beam centroid, defined as the first-order moments, and sized three times the beam widths in the x and y directions. Since the beam centroid coordinates and beam widths are as yet unknown, the procedure starts with an initial guess of the integration area. This initial integration area should approximate the beam extent and position. Using this integration area, initial values for the beam position and size are obtained which are used to recalculate the integration area. From the new integration area new values for the beam position and size are calculated. This procedure shall be repeated until the convergence of the result is obtained.

For a simple astigmatic beam the orientation of its principal axes, or the azimuthal angle  $\varphi$ , can be obtained from the second-order moments of the power density distribution by

$$\varphi(z) = \frac{1}{2} \arctan\left(\frac{2\sigma_{xy}^2}{\sigma_x^2 - \sigma_y^2}\right)$$
(15)

for  $\sigma_x^2 \neq \sigma_y^2$ . Otherwise, the azimuthal angle  $\varphi$  is obtained by

$$\varphi = \operatorname{sgn}(\sigma_{xy}^2) \frac{\pi}{4}$$
(16)

where

$$\operatorname{sgn}(\sigma_{xy}^{2}) = \frac{\sigma_{xy}^{2}}{\left|\sigma_{xy}^{2}\right|}$$
(17)

The beam widths in the direction of its principal axes are given by

$$d_{\sigma x}(z) = 2\sqrt{2} \left\{ \left( \sigma_x^2 + \sigma_y^2 \right) + \gamma \left[ \left( \sigma_x^2 - \sigma_y^2 \right)^2 + 4 \left( \sigma_{xy}^2 \right)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}} \right\}^{\frac{1}{2}}$$
(18)

$$d_{\sigma y}(z) = 2\sqrt{2} \left\{ \left(\sigma_x^2 + \sigma_y^2\right) - \gamma \left[ \left(\sigma_x^2 - \sigma_y^2\right)^2 + 4 \left(\sigma_{xy}^2\right)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}} \right\}^{\frac{1}{2}}$$
(19)

where

$$\gamma = \operatorname{sgn}\left(\sigma_x^2 - \sigma_y^2\right) = \frac{\sigma_x^2 - \sigma_y^2}{\left|\sigma_x^2 - \sigma_y^2\right|}$$
(20)

for  $\sigma_x^2 \neq \sigma_y^2$ . Otherwise the beam widths are given by

$$d_{\sigma x}(z) = 2\sqrt{2} \left( \sigma_x^2 + \sigma_y^2 + 2 \left| \sigma_{xy}^2 \right| \right)^{1/2}$$
(21)

$$d_{\sigma y}(z) = 2\sqrt{2} \left( \sigma_x^2 + \sigma_y^2 - 2 \left| \sigma_{xy}^2 \right| \right)^{1/2}$$
(22)

Perform these calculations for each measurement and calculate the mean values and the standard deviations for the beam widths and the azimuth angle.

If the ellipticity,  $\varepsilon$ , is larger than 0,87, the beam profile may be considered to be of circular symmetry at that measuring location and the beam diameter can be obtained from

$$d_{\sigma}(z) = 2\sqrt{2} \left(\sigma_x^2 + \sigma_y^2\right)^{\frac{1}{2}}$$
(23)

### 8 Measurement of divergence angles

#### 8.1 Test procedure

Locate the focusing element in the beam path in such a way that its optical axis is coaxial with the laser beam to be measured.

Locate the measuring plane of the detector system one focal length, *f*, away from the rear principal plane of the focusing element.

NOTE In general, this location is not identical with the waist location behind the focusing element.

Perform at least five measurements of the beam widths  $d_{\sigma xf}$ ,  $d_{\sigma yf}$  or the beam diameter  $d_{\sigma f}$  at that location in accordance with Clause 7.

#### 8.2 Evaluation

Calculate the divergence angle(s) of the unfocused beam according to the equations given in 5.3 for each measurement and calculate the mean value(s) and the standard deviation(s) for the divergence angle(s).

### 9 Combined determination of beam waist locations, beam widths, divergence angles and beam propagation ratios

If the beam waist is accessible for direct measurement, the beam waist location, beam widths, divergence angles and beam propagation ratios shall be determined by a hyperbolic fit to different measurements of the beam width along the propagation axis *z*. Hence, measurements at at least 10 different *z* positions shall be taken. Approximately half of the measurements shall be distributed within one Rayleigh length on either side of the beam waist, and approximately half of them shall be distributed beyond two Rayleigh lengths from the beam waist. For simple astigmatic beams this procedure shall be applied separately for both principal directions.

A preliminary test for general astigmatism shall be applied to the measured data. For each measured profile, the beam widths  $d_{\sigma x}$  and  $d_{\sigma y}$  and the azimuth angle,  $\varphi$  with respect to the laboratory system shall be calculated. If the difference in the azimuth angle of any two non-circular profiles is greater than 10° the beam shall be considered as general astigmatic and ISO 11146-2 shall be applied.

NOTE 1 Failure of this test is not proof of stigmatism or simple astigmatism. The beam may suffer from hidden general astigmatism, which can be detected by the procedures given in ISO 11146-2.

The hyperbolic fit to the measured diameters,  $d_{\sigma}$  along the propagation distance, *z*, can be expressed in the following way:

$$d_{\sigma}(z) = \sqrt{a + bz + cz^2} \tag{24}$$

The coefficients a, b, c (or  $a_x$ ,  $a_y$ ,  $b_x$ ,  $b_y$ ,  $c_x$ ,  $c_y$ ) of the hyperbola(e) shall be determined by appropriate numerical or statistical curve-fitting techniques (see Notes 2 and 3). The values of the beam propagation parameters can be obtained using:

$$z_0 = \frac{-b}{2c} \tag{25}$$

$$d_{\sigma 0} = \frac{1}{2\sqrt{c}} \sqrt{4ac - b^2} \tag{26}$$

$$\Theta_{\sigma} = \sqrt{c} \tag{27}$$

$$z_{\mathsf{R}} = \frac{1}{2c} \sqrt{4ac - b^2}$$
(28)

$$M^2 = \frac{\pi}{8\lambda} \sqrt{4ac - b^2}$$
(29)

NOTE 2 If more than one diameter measurement is performed at each z position, it is advisable to weight the data points in an inversely proportional manner to the variance of the data points.

NOTE 3 The fit should be performed by minimizing the sum of the squared relative deviations of the diameters.

NOTE 4 Astigmatic waist separation  $\Delta z_a$ , which is also known as astigmatic difference, is given by

$$\Delta z_a = \left| z_{0x} - z_{0y} \right|$$

[see 3.3.4 of ISO 15367-1:2003].

If the beam waist is not accessible for direct measurement, the same procedure shall be applied to an artificial waist created by using an aberration-free focusing element as defined in 6.6. According to Figure 3, the distances  $z_{0,2}$  or  $z_{0,2x}$  and  $z_{0,2y}$  from the artificial waist to the rear principal plane H<sub>2</sub> of the focusing element and the beam widths  $d_{\sigma 2}$  or  $d_{\sigma 2x}$  and  $d_{\sigma 2y}$  shall be determined at the artificial waist. From these data, the waist location(s)  $z_{0,1}$  or  $z_{0,1x}$  and  $z_{0,1y}$  of the original beam with respect to the front principal plane H<sub>1</sub> of the focusing element, can be calculated using

$$z_{0,1} = V^2 x_2 + f \tag{30}$$

where  $x_2$  (or  $y_2$ ) is determined using

$$x_2 = z_{0,2} - f \tag{31}$$

and where

$$V = \frac{f}{\sqrt{z_{\rm R2}^2 + x_2^2}}$$
(32)

and where

*f* is the focal length of the lens;

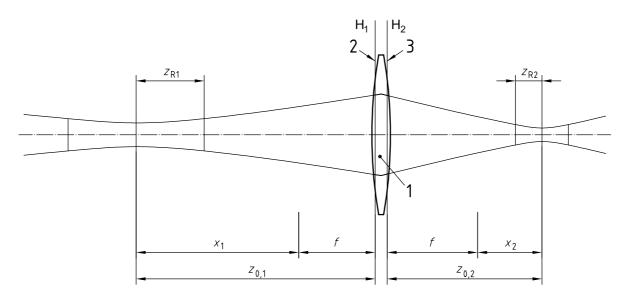
 $z_{R2}$  is the Rayleigh length of the artificial beam waist.

The Rayleigh length of the artificial waist  $z_{R2}$  can be determined by using the equations (24) and (28) for the hyperbolic fit procedure.

(33)

The beam waist diameter or widths can be calculated in the following way:

$$d_{\sigma 1} = V \cdot d_{\sigma 2}$$



#### Key

- 1 focusing element
- 2 front principal plane H<sub>1</sub>
- 3 rear principal plane H<sub>2</sub>

Figure 3 — Scheme for calculation of beam waist location(s)

The Rayleigh length can be calculated in the following way:

$$z_{\mathsf{R1}} = V^2 \cdot z_{\mathsf{R2}} \tag{34}$$

The beam divergence can be calculated in the following way:

$$\Theta_1 = \frac{\Theta_2}{V} \tag{35}$$

### 10 Test report

The following information shall be included in the test report:

### a) General information

- 1) test has been performed in accordance with ISO 11146-1:2004;
- 2) date of test;
- 3) name and address of test organization;
- 4) name of individual performing the test.

### b) Information concerning the tested laser

- 1) laser type;
- 2) manufacturer;
- 3) manufacturer's model designation;
- 4) serial number.

### c) Test conditions

- 1) laser wavelength(s) at which tested;
- 2) temperature in K (diode laser cooling fluid) (only applicable for diode lasers);
- 3) operating mode (cw or pulsed);
- 4) laser parameter settings:
  - i) output power or energy,
  - ii) current or energy input,
  - iii) pulse energy,
  - iv) pulse duration,
  - v) pulse repetition rate;
- 5) polarization;
- 6) environmental conditions.

### d) Information concerning testing and evaluation

- 1) evaluation method used:
  - i) second order moment
  - ii) alternative method: variable aperture (in accordance with ISO/TR 11146-3)

- iii) alternative method: moving knife edge (in accordance with ISO/TR 11146-3)  $\Box$
- iv) alternative method: moving slit (in accordance with ISO/TR 11146-3)
- 2) test equipment:
  - i) camera 🛛
  - ii) moving pinhole  $\Box$
  - iii) moving knife edge
  - iv) moving slit
  - v) variable aperture  $\Box$
  - vi) filter
- 3) detector and sampling system:
  - i) response time of the detector system,
  - ii) trigger delay of sampling (for pulsed lasers only),
  - iii) measuring time interval (for pulsed lasers only);
- 4) beam forming optics and attenuating method:
  - i) type of attenuator,
  - ii) type of beam splitter,
  - iii) type of focusing element;
- 5) other optical components and devices used for the test (polarizer, monochromator, etc.);
- 6) other relevant parameters or characteristics of the test which shall be chosen (aperture setting, reference plane, reference axis, laboratory system).

### e) Test results

1) Beam widths or beam diameter and azimuth angle (in accordance with Clause 7)

Location z

	Mean value	Standard deviation
Beam diameter $d_{\sigma}$		
Beam width $d_{\sigma x}$		
Beam width $d_{\sigma y}$		
Azimuth angle $\varphi$		

2) Beam divergence angles (in accordance with Clause 8)

Focusing element used\_\_\_\_\_

Focal length \_\_\_\_\_

	Mean value	Standard deviation
Beam divergence angle $ \varTheta_{\sigma} $		
Beam divergence angle $ \varTheta_{\sigma \! x} $		
Beam divergence angle $ \varTheta_{\sigma \! y} $		

3) Beam propagation parameters derived from hyperbolic fit (in accordance with Clause 9)

	Value	Estimated uncertainty
Beam waist location $z_0$		
Beam waist location $z_{0x}$		
Beam waist location $z_{0y}$		
Beam waist diameter $d_0$		
Beam waist width $d_{0x}$		
Beam waist width $d_{0y}$		
Azimuth angle $\varphi$		
Rayleigh length $z_R$		
Rayleigh length $z_{Rx}$		
Rayleigh length $z_{Ry}$		
Beam divergence angle $ \varTheta_{\sigma} $		
Beam divergence angle $ arPhi_{\sigma \! x} $		
Beam divergence angle $ {\cal O}_{\sigma \! y} $		
Beam propagation ratio $M^2$		
Beam propagation ratio $M_x^2$		
Beam propagation ratio $M_y^2$		

### Bibliography

- [1] ISO/TR 11146-3, Lasers and laser-related equipment Test methods for laser beam widths, divergence angles and beam propagation ratios Part 3: Intrinsic and geometrical laser beam classification, propagation and details of test methods
- [2] ISO 15367-1:2003, Lasers and laser related equipment Test methods for determination of the shape of a laser beam wavefront Part 1: Terminology and fundamental aspects